The 5th Workshop on Geometric Group Theory

Days : 5 March – 8 March, 2019

Place : Okinawaken Seinenkaikan
2-15-23 Kume, Naha, Okinawa
http://www.okiseikan.or.jp/

Yoshito Ishiki (University of Tsukuba)
Tsukasa Ishibashi (The University of Tokyo)
Mao Okada (The University of Tokyo)
Morimichi Kawasaki (RIMS, Kyoto University)
Takuya Katayama (Hiroshima University)
Motoko Kato (The University of Tokyo)
Yuhei Suzuki (Nagoya University)
Kenshiro Tashiro (Kyoto University)
Takahiro Matsushita (University of the Ryukyus)
Hirokazu Maruhashi (The University of Tokyo)
Kohei Yamamoto (Tohoku University)
Xiaobing Sheng (The University of Tokyo)
Robert Tang (OIST)
Ueda Taishu (Saitama University)
Yuki Kobatake (Ehime University)
Rui Nishioka (Ehime University)
Keita Fukushima (Ehime University)

Schedule

	5 Tue	6 Wed	7 Thu	8 Fri
9:15 ∼ 10:15		Sheng	Yamamoto	Kawasaki 3
10:30 ∼ 11:30		Kawasaki 1	Katayama	Maruhashi
11:45 ∼ 12:45		Tashiro	Okada	Free Discussion
12:45 ∼ 14:30	Opening (14:15∼)	Lunch	Lunch	
14:30 ∼ 15:30	Kato	Suzuki	Kawasaki 2	
15:45 ∼ 16:45	Ishibashi	Tang	Matsushita	
17:00 ∼ 18:00	Kobatake (15 min) Ueda (15 min)	Fukushima (15 min) Nishioka(15 min)	Ishiki (40 min)	

Organizers

Tomohiro Fukaya (Tokyo Metropolitan University)
Shin-ichi Oguni (Ehime University)
Motoko Kato (The University of Tokyo)

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第 5 回幾何学的群論ワークショップ

日程 ：2019 年 3 月 5 日 – 3 月 8 日

会場 ：沖縄青年会館

沖縄県那覇市久米 2-15-23

http://www.okiseikan.or.jp/

伊敷喜斗（筑波大学）
石橋 典（東京大学）
岡田真央（東京大学）
川崎盛通（東京大学）
片山拓弥（広島大学）
加藤本子（東京大学）
鈴木悠平（名古屋大学）
田代賢志郎（京都大学）
松下尚弘（琉球大学）
丸橋広和（東京大学）
山本航平（東北大学）
Xiaobing Sheng（東京大学）
Robert Tang（OIST）
植田大秀（埼玉大学）
小畑祐貴（愛媛大学）
西岡瑠偉（愛媛大学）
福島圭太（愛媛大学）

予定表

	5 日（火）	6 日（水）	7 日（木）	8 日（金）
9:15 ～ 10:15		Sheng	山本	川崎 3
10:30 ～ 11:30		川崎 1	片山	丸橋
11:45 ～ 12:45		田代	岡田	自由討論
12:45 ～ 14:30	Opening (14:15～)	昼食	昼食	
14:30 ～ 15:30	加藤	鈴木	川崎 2	
15:45 ～ 16:45	石橋	Tang	松下	
17:00 ～ 18:00	小畑（15 分） 植田（15 分）	福島（15 分） 西岡（15 分）	伊敷（40 分）	

世話人

深谷友宏 (首都大学東京)，尾國新一（愛媛大学），加藤本子（東京大学）

本ワークショップは科学研究費・若手（B）（15K17528）同（16K17595）の援助により開催されます.

Yoshito Ishiki

Quasi-symmetric maps between one-point compactifications of monotone sequences

The class of quasi-symmetric maps is studied in geometric analysis on metric spaces. This class contains bi-Lipschitz maps and homogeneously bi-Hölder maps. In this talk, a one-point compactification of a monotone sequence means a metric space consisted of a monotone sequence and its limit point. We introduce studies on quasi-symmetric maps between them.

Tsukasa Ishibashi

Cluster realizations of Coxeter groups

Cluster modular group is the automorphism group of a cluster algebra. It also acts on several geometric objects, called the cluster varieties. An embedding of a group into a cluster modular group provides us with a uniform combinatorial description of that group and possibly new representations (e.g. quantum representations of MCGs). In this talk, we construct embeddings of a Coxeter group associated with Kac-Moody Lie algebras into several cluster modular groups. Some of them have connections with the theories of integrable systems and higher Teichmuller spaces. My talk is based on a joint work with Rei Inoue and Hironori Oya.

Mao Okada

Local rigidity of certain actions of nilpotent-by-cyclic groups on the boundary of rank one symmetric spaces.

In 2012, Asaoka constructed a conformal action on the sphere with a common fixed point which exhibits a kind of local rigidity. The action is a one-point compactification of an action on the Euclidean space which is the extension of an abelian action of translations by a cyclic action of integer multiplications. Viewing a conformal transformation of the sphere as the extension to the boundary of an isometry of the real hyperbolic space, we generalized the result to actions on the boundary of rank one symmetric spaces of non-compact type. The actions are one-point compactifications of actions on nilpotent Lie groups which are the extensions of nilpotent actions of translations by cyclic actions of dilations.

Morimichi Kawasaki

Series of lectures on quasimorphisms

First talk: Introduction to quasimorphisms

In the first talk, I introduce the notion of quasimorphisms. I review the definition and foundamental properties of quasimorphisms. And I explain some motivations of quasimorphisms from the views of the bounded cohomology and hyperbolic geometry.

Second talk: Stable commutator length and Bavard's duality theorem

In this talk, I introduce stable commutator length. I explain Bavard's duality theorem which states that stable commutator length and quasimorphism have a deep relation. This theorem has been generalized by Calegari-Zhuang, Kawasaki and Kimura. I explain the proof of Bavard's duality theorem along their starategies as time permits.

Third talk: Quasimorphisms in symplectic geometry

In this talk, I explain applications of quasimorphisms to symplectic geometry. Our goal is proving Entov-Polterovich's theorem which states that any integrable system on a closed symplectic manifold admits a non-displaceable fiber. To prove that theorem, they construct "Calabi quasimorphism" and use their quasimorphism property.

Takuya Katayama

On virtual embeddings of the braid groups into the mapping class groups of surfaces

A classical theorem due to Birman-Hilden implies that the braid group B_{2g} on $2g$ strands is embedded in the mapping class group $\mathrm{Mod}(S_g)$ of a closed orientable surface of genus g. In this talk, by investigating a certain class of right-angled Artin groups in mapping class groups, we will prove that no finite index subgroup of B_{2g+1} is embedded in $\mathrm{Mod}(S_g)$. This talk is based on joint work with Erika Kuno.

Motoko Kato

Group actions on finite dimensional non-positively curved spaces

We give a sufficient condition for groups to have a global fixed point whenever they act semi-simply on a finite dimensional Busemann space. As an application, we show that various finitely generated simple subgroups of the homeomorphism group of the circle, in particular Richard Thompson's groups, have such a property.

Yuhei Suzuki

Complete descriptions of intermediate operator algebras by intermediate extensions of dynamical systems

Topological dynamical systems (t.d.s.=a discrete group action on a locally compact Hausdorff space) provide a rich source of interesting C*-algebras via the crossed product construction. The subject of this talk is on the following functoriality of the crossed product construction: each proper factor (=equivariant proper quotient map on a t.d.s.) associates an inclusion of the crossed product C*-algebras. The main theorem shows that under the freeness assumption, ALL intermediate C*-algebras in fact come from the intermediate quotient. This provides variously many new examples of C*-algebra inclusions whose all intermediate C*-algebras are described. As an application, we obtain a new realization result of intermediate operator algebra lattice. Based on arXiv:1805.02077.

Kenshiro Tashiro

On the word growth function of nilpotent groups

According to the Gromov's polynomial growth theorem, the word growth function of (Γ, S), denoted by $b : \mathbb{N} \to \mathbb{N}$, is bounded by polynomial iff Γ is virtually nilpotent. It is natural to ask what polynomial approximates b at the infinity. Namely, our interest is the optimal

$0 < \alpha \le 1$ such that

$$b(n) = cn^d + O(n^{d-\alpha}) \quad (n \to \infty).$$

Here the constants c and d are already known, and written by geometric data of the asymptotic cone of the Cayley graph $Cay(\Gamma, S)$.

In 1998, Stoll showed that if the nilpotency class is not greater than 2, then $\alpha = 1$. For the greater classes, Breuillard and Le Donne gave a sufficient condition for α being 1. The condition consists of two parts. One is on approximatability of $Cay(\Gamma, S)$ to a certain sub-Finsler nilpotent Lie group, and the other is on the rectifiability of the sphere of the asymptotic cone.

I will talk about recent progress of the latter part, where one uses techniques of optimal control theory.

Takahiro Matsushita

Fundamental groups of neighborhood complexes

The neighborhood complex is a simplicial complex associated to a graph, which was introduced in the graph coloring problem.

For a based graph (G, v) and a positive integer r, we introduce the r-fundamental group $\pi_1^r(G, v)$. This group is obtained as a quotient of the set of loops of (G, v) with respect to some equivalence relation. The r-fundamental group has a natural subgroup $\pi_1^r(G, v)_{ev}$, called the even part, and we show that the even part $\pi_1^2(G, v)_{ev}$ of the 2-fundamental group is isomorphic to the fundamental group $\pi_1(N(G), v)$ of the neighborhood complex.

We apply the r-fundamental groups to the existence problem of graph homomorphisms. Our principal application is to show the non-existence of graph homomorphisms from Kneser graphs $KG_{2k+1,k}$ to the 5-cycle graph C_5. Moreover, we introduce r-covering maps as graph homomorphisms satisfying certain properties, and show that there is a close relationship between r-fundamental groups and r-covering maps, as is the case of fundamental groups and covering spaces in topology.

Hirokazu Maruhashi

Computation of the de Rham cohomology of certain foliations

It's quite straightforward to define the de Rham cohomology of a smooth foliation of a manifold if you know the definition of the de Rham cohomology of a manifold. What's not straightforward is its computation. It's been more than four decades since this concept was first investigated with regard to the deformation theory of foliations, but not much computations have been done to this day in my opinion. (Probably because people haven't paid much attention to it.) For example, let $\mathcal{F}$ be the weak stable foliation of the geodesic flow of a compact hyperbolic surface, which is a good example of a foliation you can find in any textbook on foliations. The first cohomology of $\mathcal{F}$ was computed by Matsumoto and Mitsumatsu in 2003. But the second cohomology has remained unknown as far as I know. In this talk I will show you how to compute the cohomology of $\mathcal{F}$ in all degrees at the same time. A Kodaira–Spencer theory for parameter deformations of a locally free action was a motivation for the computation. This is a joint work with Mitsunobu Tsutaya from Kyushu University.

Kohei Yamamoto

Percolation and Triangle condition

Percolation is a process to construct a probability measure on the set of subgraphs for a graph. For a given parameter p, each edge will be open with probability p, and closed with probability 1-p independently. We construct a subgraph with all open edges. Each connected component of a subgraph is referred to as cluster. One of important topics in percolation is whether an infinite cluster exists or not. For a given graph, there exists a threshold which called critical probability. The critical probability is related to the expected value of a size of cluster containing a fixed point. This expected value is a non-decreasing function in p and diverging at the critical point. Aizenman and Newman introduced the triangle condition to analyze the asymptotic behavior of the expected value. In this talk, we consider percolation on the product graph of regular tree and a line. I speak my result on the triangle condition.

Xiaobing Sheng

Quasi-isometrically embedded subgroups T_n of Thompson's group T

Brown has defined the generalised Thompson's group F_n, T_n, where n is an integer at least 2 and Thompson's groups $F = F_2$ and $T = T_2$ in the 80's. Burillo, Cleary and Stein have found that there is a quasi-isometric embedding from F_n to F_m where n and m are positive integers at least 2. We show that there is a quasi-isometric embedding from T_n to T_2 for any $n \leq 2$ and no embeddings from T_2 to T_n for $n \leq 3$. In the talk, I will be briefly introduce Thompson's groups and outline the proof. I will also mention some related work from a geometric point of view.

Robert Tang

Cubical geometry in the polygonalisation complex

In this talk, I will introduce the polygonalisation complex, a cube complex associated to a surface with marked points. We show that this complex has a hyperplane structure reminiscent of those in CAT(0)-cube complexes, despite not being CAT(0) itself. We use this hyperplane structure to prove that any isomorphism between polygonalisation complexes is induced by a homeomorphism between the associated surfaces. This is joint work with Mark Bell and Valentina Disarlo.

Taishu Ueda

Definition of geodesic space having controlled product

In this lecture, I want to construct a graph space X' (space constructed by vertexes and edges) in which we can isometrically embed a metric space X and want to be controlled product in X'. The motivation is as below. In 2017, it is proved in (1) that the coarsely convex space satisfy the coarse Baum-Connes Conjecture. In the proof, key factors are compactification by

Gromov product, quasi geodesic, radial contraction, and so on. On the other hand, in 2018, (pre)controlled product is defined in (2) and it is proved that there is a bijective correspondence between all controlled products and all coarse compactifications. However, in the argument, there is no (quasi)geodesic on the spaces. Hence, I try to combine the concept of geodesic and controlled product space for applying technique used in (1) to spaces handled in (2). I want to find out new spaces satisfying coarse Baum-Connes Conjecture.

(1) Tomohiro Fukaya and Shinichi-Oguni, A coarse Cartan-Hadamard Theorem with application to the coarse Baum-Connes Conjecture arxiv:1705.05588v3

(2)Tomohiro Fukaya and Shin-ichi Oguni and Takamitsu Yamauchi, Coarse compactifications and controlled products arxiv:1810.08720v1

Yuki Kobatake

On hyperbolicity of combinatorial horoballs (Japanese)

Combinatorial horoballs were introduced by Groves-Manning. These are used in studying relatively hyperbolic groups. In this talk, we will focus on properties as metric spaces of combinatorial horoballs.

Rui Nishioka

The least number of charts for a Euclidean structure (Japanese)

Any N-dimensional torus is known to admit Euclidean structures. For a Euclidean structure on the 2-dimensional torus, we will see that the least number of charts among compatible atlases is 3. In the proof we will use the Lusternik-Schnirelmann category.

Keita Fukushima

On nonpositive curvedness of Hilbert geometries (Japanese)

Bounded convex domains with Hilbert metrics are called Hilbert geometries. In this talk, we will discuss whether Hilbert geometries are nonpositively curved or not.