Group actions and K-theory

Day : March 12, 2012 – March 15
Place : Department of Mathematics, Kyoto University
Room 110
http://www.math.kyoto-u.ac.jp/%7Etomo/g-and-k/

Abstracts

Shin-ichi Oguni (Ehime university)

Blowing up and down compacta with geometrically finite convergence actions of a group

The notion of relatively hyperbolic groups was introduced by Gromov, which is a generalized notion of fundamental groups of noncompact complete hyperbolic manifolds of finite volume. Relative hyperbolicity for a group is deeply related to geometrically finite convergence actions of the group. In this talk I will present an introduction to relatively hyperbolic groups and discuss relation among geometrically finite convergence actions of a fixed group. This talk is based on a joint work with Yoshifumi Matsuda and Saeko Yamagata.

Tomohiro Fukaya (Kyoto university)

The coarse Baum-Connes conjecture for relatively hyperbolic groups

The Baum-Connes conjecture is a part of Connes' non-commutative geometry program. It can be viewed as a conjectural generalization of the Atiyah-Singer index theorem, to the equivariant setting, that is, the ambient manifold is not compact, but some compactness is restored by means of proper co-compact action. Higson and Roe gave a further generalization of those conjectures to coarse geometry setting. Coarse geometry is the study of metric spaces from a 'large scale' point of view.

Like the Atiyah-Singer theorem, the coarse Baum-Connes conjecture states that a topological object coincides with a purely analytic one. For given metric space X, the topological object is the coarse K-homology of X, while the analytic object is the K-theory of the C^*-algebra associated with the space X, which is called Roe algebra.

The coarse Baum-Connes conjecture implies several other classical conjectures, for example, the Gromov-Lawson conjecture on the non-existence of a

positive scalar curvature, the Novikov conjecture on the homotopy invariance of the higher signatures.

In this talk, I will report our results on the coarse Baum-Connes conjecture for a group which is hyperbolic relative to a finite family of infinite subgroups. We showed that the group satisfies the coarse Baum-Connes conjecture if each subgroup belonging to the family satisfies the coarse Baum-Connes conjecture and admits a finite universal space for proper actions. If the group is torsion-free, then it satisfies the analytic Novikov conjecture. This is a joint work with Shin-ichi Oguni.

Otgonbayer Uuye(Cardiff university)

Restriction Maps in Equivariant KK-theory

Cyclic groups play a distinguished role in the representation theory of compact Lie groups: any virtual representation which restricts trivially to every finite cyclic subgroup is itself trivial. Jackowski and McClure studied how far this extends to virtual equivariant vector bundles i.e. to equivariant K-theory. In this talk we explain how to extend their result to finite $RO(G)$-gradable modules over K-theory. The key tool here is an extension of the completion theorem of Adams-Haeberly-Jackowsi-May.

Yoshitaka Kida (Kyoto university)

Invariants for orbit equivalence relations of Baumslag-Solitar groups

For non-zero integers p, q, the Baumslag-Solitar group BS(p, q) is defined by the presentation $\langle a, t \mid ta^p t^{-1} = a^q \rangle$. We consider ergodic free probability-measure-preserving actions of BS(p, q), and introduce an invariant of them under orbit equivalence. We also present results on groups which are measure equivalent to BS(p, q).

Makoto Yamashita (Ochanomizu university)

Deformation of algebras associated with group cocycles

Fell bundles over a discrete group can be deformed using 2- cocycles on the base group. We give a K-theoretic isomorphism of such deformations, generalizing the previously known cases of the theta- deformations and the reduced twisted group algebras. We shall also discuss the pairing of invariant cyclic cocycles with the K-groups of deformed algebras.

Hang Wang (Tsinghua university)

On Kasparov's K-theoretic index formulas

Generally speaking, index formulas express analytical informations of a system of elliptic equations, or an elliptic operator, on some manifold in terms of the topology and the global geometry of the manifold. When X is a complete Riemannian manifold and G is a locally compact group acting on X properly, cocompactly and isometrically, a G-invariant elliptic operator have an index in the K-theory of the maximal group C^*-algebra of G. This K-theretic index is an example supplied by the Baum-Connes conjecture. Kasparov's index formula express this index as a pairing of its symbol and a Dolbeault operator. In a special case when G is trivial and when X is compact, the formula reduces to the Atiyah-Singer index formula when applying the Chern Character to the K-theoretic index. In the first half of my talk, I will discuss how to use Kasparov index formula to derive a formula for a real-valued L^2-index, defined using G-trace (interpreted as the 0-degree of some Chern character map), of a G-invariant operator. In the second part, I will talk about Kasparov's work on K-theoretic index formula for transversal elliptic operators on X, where G is a Lie group acting on properly isometrically on X. Leading by the interest in looking for cohomological formulas of such operators, I will survey the known results in some special cases, time permitted.

Masayoshi Matsumura (University of Tokyo)

C^*-algebraic characterization of amenable actions

A. Hulanicki proved in 1966 that the amenability of a group is equivalent to coincidence of its full and reduced C^*-algebras, that is, weak containment of every unitary representation in the regular representation. C. Anantharaman-Delaroche proved in 1987 the amenability of a group action on a compact space implies coincidence of the associated full and reduced C^*-algebras. The converse remains open. We will give a partial answer of this problem introducing equivariant variants of the weak expectation property (WEP) of C. Lance and the property C" of R. J. Archbold and C. J. K. Batty.

Shouhei Honda (Kyushu university)

A note on one dimensional regular sets

Let Y be the Gromov-Hausdorff limit space of a sequence of complete Riemannian manifolds with a lower Ricci curvature bound. In this talk, we will show that if the one dimensional regular set of Y is not empty, then Y is a complete one dimensional Riemannian manifold.

Yurhiko Umemoto (Osaka City university)

On the growth series of 3-dimensional hyperbolic Coxeter groups with 4 and 5 generators

In general, if we consider a group G with finite generating set S, we obtain the length function on G with respect to S. Then the formal power series whose coefficients are determined by the length function, called the growth series is defined. For the case of a hyperbolic Coxeter group which is a discrete group generated by reflections with respect to facets of a hyperbolic Coxeter polyhedron, it is known that the growth series is a rational function. In this talk, I will present the growth series of hyperbolic Coxeter groups with respect to 3-dimensional hyperbolic simplices and pyramids, i.e., with 4 and 5 generators, and study arithmetic properties of the growth rates of them.

This is a joint work with Yohei Komori (OCAMI and Osaka City University).

Qinggnag Ren (Chongqing university)

Coxeter group and aspherical manifold

In this talk, I will talk about the aspherical manifolds constructed by M.W.Davis using reflection technique. The fundamental groups of those manifolds are subgroups of some Coxeter group. Thus the subgroup structure of the Coxeter group has a close relation to the aspherical manifold. This also provide us a new viewpoint to the structure of surface groups and other questions.

Kei Funano (Kyoto university)

Infinite and l_p versions of nonlinear Dvoretzky's theorem

Roughly speaking, Dvoretzky's theorem asserts that every metric space of sufficiently large size contains a subset of large size that embeds into Hilbert space. This can be considered as a Ramsey type theorem. In this talk I will discuss my recent results about Dvoretzky's theorem.

Otgonbayar Uuye (Cardiff university)

The Kunneth theorem for K-theory

The Kunneth theorem for K-theory computes the K-theory of a tensor product of C*-algebras. For non-nuclear algebras, it has long been known that one has to distinguish the maximal and the minimal tensor products when applying the Kunneth theorem. In this talk, we show that there are counter-examples to the

Kunneth theorem for both tensor products. In fact, we show that there are examples satisfying the Kunneth theorem for one tensor product but not for the other, in both cases.

Takumi Yokota (Kyoto university)

On the filling radius of positively curved Alexandrov spaces

It was shown by F. Wilhelm that Gromov's filling radius of any positively curved closed Riemannian manifolds are less than that of the round sphere unless they are isometric to each other. In this talk, I will talk about its generalization to closed Alexandrov spaces of positive lower curvature bound.

Masato Mimura (University of Tokyo)

Property $(TT)/T$ and homomorphism superrigidity into mapping class groups

Mapping class groups (**MCG**'s), of compact oriented surfaces (possibly with punctures), have many mysterious features: they behave not only like higher rank lattices (namely, irreducible lattices in higher rank algebraic groups); but also like rank one lattices. The following theorem, the *Farb–Kamanovich–Masur superrigidity*, states a *rank one phenomenon* for MCG's:

> *Every group homomorphism from higher rank lattices* (*such as* $\mathrm{SL}(3,\mathbb{Z})$ *and cocompact lattices in* $\mathrm{SL}(3,\mathbb{R})$) *into MCG's has* **finite** *image.*

In this talk, we show a generalization of the superrigidity above, to the case where higher rank lattices are replaced with some (*non-arithmetic*) *matrix groups over general rings*. Our main example of such groups is called the "**universal lattice**", that is, the special linear group over commutative finitely generated polynomial rings over integers, (such as $\mathrm{SL}(3,\mathbb{Z}[x])$). To prove this, we introduce the notion of "*property* (TT)/T" for groups, which is a strengthening of *Kazhdan's property* (T).

We will explain these properties and relations to ordinary and bounded cohomology of groups (with twisted unitary coefficients); and outline the proof of our result.

The preprint on this talk is available at: **arXiv:1106.3769**

Atsushi Yamashita (Tohoku university)

Metric compactifications and coarse structures

This is a joint work with Kotaro Mine. After reviewing the notion of coarse space by John Roe, I will introduce our results which relate the C_0 coarse structure on a totally bounded, locally compact metric space and the topology of its Higson corona. This can be regarded as an analogue of the correspondence between Gromov hyperbolic groups and their boundaries.

Takefumi Kondo (Kobe university)

Poincare inequality and fixed point property of isometric group actions

Hokuto Morishima (Osaka university)

Algebraic translation algebras

We introduce an algebraic version of the translation algebra of a group. A quasi-isometry of two finitely generated groups induces Morita equivalence of their algebraic translation algebras. Every Morita invariant is a quasi-isometry invariant. We show some examples of the algebraic quasi-isometry invariants. In this talk mainly, we prove converse of the basic theorem above in the case of R-coefficient: Morita equivalence of the algebraic translation algebras with R-coefficient of two finitely generated groups induces a quasi-isometry of these groups.

Jeremie Brieussel (Kyoto university)

Behaviors of entropy on finitely generated groups

I will present a family of finitely generated groups, for which the entropy of a simple random walk can be computed. This provides in particular a continuum of entropy behaviors, as well as oscillation phenomena. These groups are realized by automorphisms of spherically homogeneous rooted trees.

Hiroki Sako (Kyoto university)

Yu's property A and operator norm localization property for metric spaces with bounded geometry

I will show that operator norm localizaion property for metric spaces with bounded geometry is equivalent to Yu's property A. I will also make comments on nuclearity, completely bounded approximation property and exactness on uniform Roe algebra.